Math Club Problem Set #2 CIMC PREP

1. Given that $f(x) = 2x+1$ and $g(x) = 3x+1$, determine the value of $f(g(3))$

2. Given that $f(x) = 1-x$ and $g(x) = \dfrac{x}{1-x}$ where $x \neq 1$.

   I) Prove that $g(f(x)) = \dfrac{1}{g(x)}$.

   Ii) Does $f(g(x)) = \dfrac{1}{f(x)}$

3. Given that $f(x) = ax+b$, $g(x) = cx+d$, and $f(g(x)) = g(f(x))$, how are "a", "b", "c", and "d" related?

4. Suppose that for all $x > 0$ we have $f(2x) = \dfrac{5}{2+x}$. What is $2f(x)$?

5. Find all functions $f(x)$ such that $f(x+2y) + f(x-2y) = 2x^2 + 8y^2$ for all real "x" and "y'

6. The function "f" is such that $f(0)=0$ and $f(2x+1)=2f(x)+3$ for all real "x". Find $f(1)$, $f(3)$, and $f(7)$

7. $F(n)$ is a function such that $F(1)=F(2)=F(3)=1$ and $F(n+1)=\dfrac{F(n)F(n-1)+1}{F(n-2)}$ for $n\geq3$. Find $F(6)$

8. The function "f" has a domain and range of [0,1]. Given that $f(0)=0$, $f(1)=1$, $f\left(\dfrac{x+y}{2}\right)=\dfrac{f(x)+f(y)}{2}$ for all $x,y\in[0,1]$. Find all possible values of $f\left(\dfrac{1}{2}\right)$ and $f\left(\dfrac{1}{3}\right)$

9. If $f(x-y)=f(x)\times f(y)$ for all "x" and "y", and $f(x)\neq0$. Find all possible values of $f(1977)$

10. Let $f(x)=\dfrac{x+1}{x+c}$. If $f(f(x))=x$ for all values of "x", find the value of "c".

11. Find $f(x)$ if $f(a+b)-f(a-b)=4ab$ for all $a,b\in\mathbb{R}$

12. Find all functions "f" such that $f(x)+f(x+y)+f(x+2y)=6x+6y$ for all real numbers "x" and "y"

13. Determine all functions that satisfy the identity: $f(xy)=\left(y^2-y+1\right)f(x)$

14. Given that a function "f" has the property that $f(x)+2f\left(\dfrac{1}{x}\right)=3x$ for all nonzero real numbers "x".

How many nonzero solutions are there to the equation $f(x)=f(-x)$.